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AN APPLICATION OF A GROUP OF ORDER 16 TO A CONFIGURATION ON AN ELLIPTIC CUBIC.*

By Arnold Emch.

1. Introduction. Let P be a point on a cubic C_3, a, b two distinct tangents from P to the C_3 with the points of tangency A and B. From A let there be a line $a_{11'}$, distinct from a, intersecting the C_3 in the points 1 and 1′. Let the joins $b_{12'}$ and $b_{1'2}$ of B with 1 and 1′ intersect the C_3 in 2′ and 2, respectively; then, as is well known, A, 2 and 2′ are on the same line $a_{22'}$. Two points A and B on a cubic are called a Steinerian† couple, when the tangents a and b at those points meet in a point of the same cubic. The cubic may now be generated by two projective quadratic pencils of lines with a self corresponding common line. Denoting this line by p and letting the letters denoting the lines also represent the left-hand sides of the linear equations of the lines, we write the equations of the two projective quadratic pencils with the parameter λ generating the cubic in the form

$$(1)\qquad \begin{aligned} m_{ab}' \cdot ap + n_{ab}' \cdot \lambda a_{11'}a_{22'} &= 0, \\ bp + \qquad \lambda b_{12'}b_{21'} &= 0, \end{aligned}$$

where m_{ab}', n_{ab}' are constant factors whose ratio is uniquely determined by the given cubic. In fact the equation of the cubic generated by (1) is

$$(2)\qquad m_{ab}' \cdot ab_{12'}b_{21'} - n_{ab}' \cdot ba_{11'}a_{22'} = 0.$$

If P is on an elliptic cubic, we can draw four tangents a, b, c, d from P to the cubic with the points of tangency A, B, C, D. With these as vertices of projective quadratic pencils of the form (1) we can generate the cubic in six different ways. Making use of the theorem‡ that a polynomial $\phi(x_1, x_2, x_3)$ contains another polynomial $\psi(x_1, x_2, x_3)$, which is irreducible, as a factor, or, except as to a constant factor, is identical with it, if ϕ vanishes for all sets of values x_1, x_2, x_3 for which ψ vanishes, from the twelve forms of the equation of the cubic resulting from those projectivities, we can establish a certain configuration of 48 lines. These are obtained from three definite ones of this configuration by the 16 substitutions of a group§ of order 16 with 15 subgroups of order 2. This group is represented by all couples of sets of four elements formed between the two groups of four sets each, i. e.,

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* Read before the American Mathematical Society at Chicago, April 21, 1916.

† Clebsch-Lindemann, Vorlesungen über Geometrie, vol. I, pp. 616—— (1876).

‡ Netto, Vorlesungen über Algebra, vol. 2, theorem IX (see also X), pp. 23–24 (1900).

§ I am indebted to Prof. G. A. Miller for information concerning this group.

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{array} \qquad\qquad \begin{array}{cccc} 1' & 2' & 3' & 4' \\ 2' & 1' & 4' & 3' \\ 3' & 4' & 1' & 2' \\ 4' & 3' & 2' & 1'. \end{array}$$

Each couple consists of two sets, one from each of these groups. All of these substitutions except the identity are of order 2. For example, from the substitution

$$S = \begin{pmatrix} 1 & 2 & 3 & 4 & 1' & 2' & 3' & 4' \\ 3 & 4 & 1 & 2 & 2' & 1' & 4' & 3' \end{pmatrix}$$

we get $S^2 = 1$.

It is the purpose of this paper to establish this configuration of 48 lines explicitly by the indicated method.

2. The twelve equations representing the cubic. Let A, B, C, D be a Steinerian quadruple formed by the points of tangency of the four tangents a, b, c, d from a point P on an elliptic cubic C_3 to this cubic. Through A draw a line intersecting C_3 in the points 1 and 1′. Let the joins $b_{12'}$ and $b_{1'2}$ of B with 1 and 1′ intersect C_3 in 2′ and 2, so that their join $a_{22'}$ passes through A (Fig.). For the sake of simplicity of the construction A may be chosen as the infinite point of the C_3, without loss of generality, so that a is an asymptote. Let the joins $c_{23'}$ and $c_{2'3}$ of C with 2 and 2′ intersect C_3 in 3′ and 3. Again the join $a_{33'}$ of 3 and 3′ will pass through A. Finally let the joins $d_{24'}$ and $d_{2'4}$ of D with 2 and 2′ intersect the C_3 in 4′ and 4, so that the join $a_{44'}$ of 4 and 4′ also passes through A. From the theory of Steinerian quadruples it is well known* that the joins $b_{34'}$ and $b_{3'4}$ of B with 3 and 3′ pass through 4′ and 4; the joins $c_{14'}$ and $c_{1'4}$ of C with 1 and 1′ pass through 4′ and 4; and the joins $d_{13'}$ and $d_{1'3}$ of D with 1 and 1′ pass through 3′ and 3, respectively. From the Steinerian quadruple A, B, C, D we can choose six couples of two points each as vertices of quadratic projectivities by which the C_3 may be generated. Each of these projectivities may be written in two different forms. Thus, the projectivity with A and B as vertices, may be written in either of the forms, which we denote by $(AB)'$ and $(AB)''$ respectively, (see (1))

$$(4) \qquad (AB)' \left\{ \begin{aligned} m_{ab}' \cdot ap + n_{ab}' \cdot \lambda a_{11'} a_{22'} &= 0, \\ bp + \lambda b_{12'} b_{1'2} &= 0, \end{aligned} \right.$$

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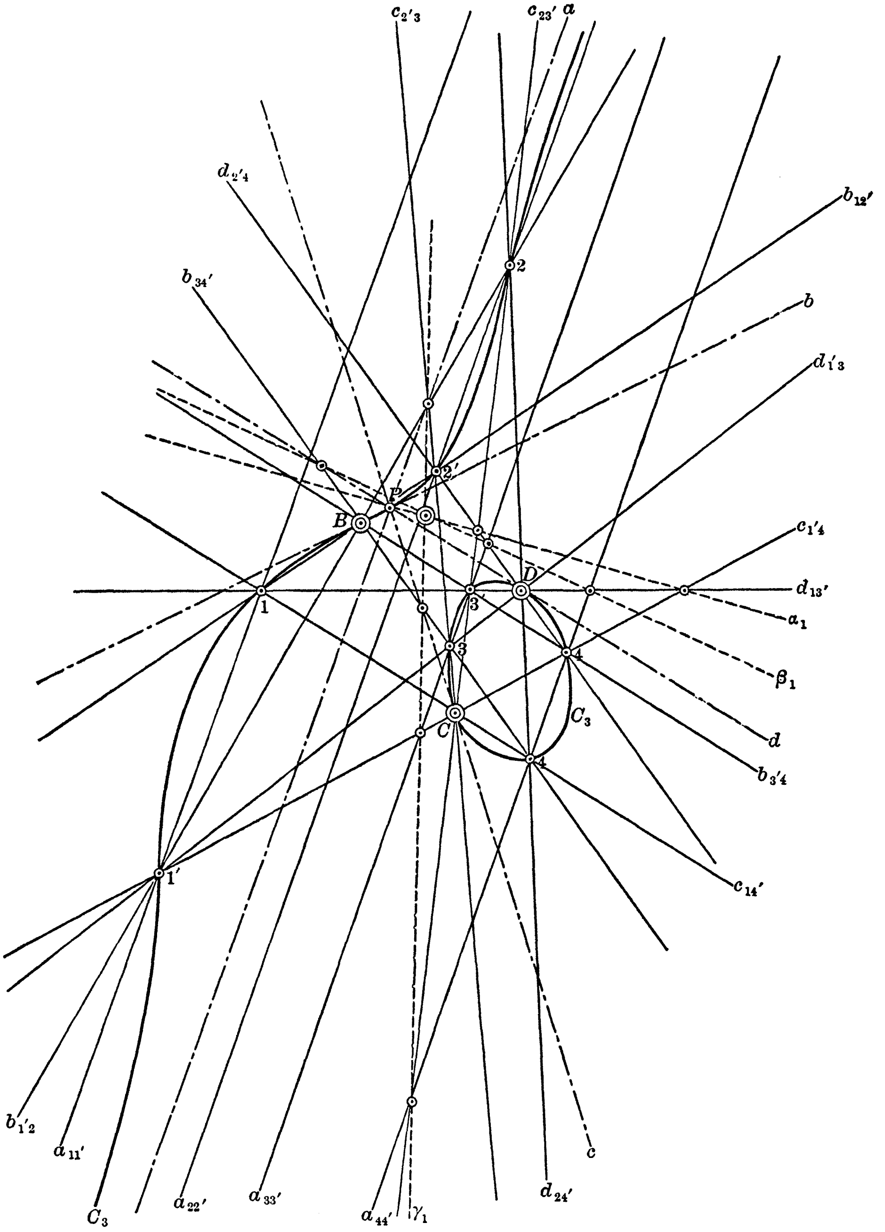
* Emch, Introduction to Projective Geometry, pp. 201–204.

and

$$(5)\qquad (AB)''\begin{cases} m_{ab}'' \cdot ap + n_{ab}'' \cdot \lambda a_{33'}a_{44'} = 0, \\ \qquad\quad bp + \qquad \lambda b_{34'}b_{43'} = 0. \end{cases}$$

Denoting the five remaining projectivities in a similar manner by $(BC)'$, $(BC)''$; $(CD)'$, $(CD)''$; etc., and forming in every case the equation gen-



erated by the projectivity, the following table of the twelve identical forms of the cubic is obtained:

$$
\begin{aligned}
(AB)' &\equiv m_{ab}' \cdot ab_{12'}b_{1'2} - n_{ab}' \cdot ba_{22'}a_{11'} = 0,\\
(AB)'' &\equiv m_{ab}'' \cdot ab_{34'}b_{3'4} - n_{ab}'' \cdot ba_{44'}a_{33'} = 0,\\
(BC)' &\equiv m_{bc}' \cdot bc_{14'}c_{2'3} - n_{bc}' \cdot cb_{34'}b_{12'} = 0,\\
(BC)'' &\equiv m_{bc}'' \cdot bc_{41'}c_{3'2} - n_{bc}'' \cdot cb_{21'}b_{43'} = 0,\\
(CD)' &\equiv m_{cd}' \cdot cd_{13'}d_{4'2} - n_{cd}' \cdot dc_{23'}c_{14'} = 0,\\
(CD)'' &\equiv m_{cd}'' \cdot cd_{31'}d_{2'4} - n_{cd}'' \cdot dc_{41'}c_{32'} = 0,\\
(AD)' &\equiv m_{ad}' \cdot ad_{31'}d_{3'1} - n_{ad}' \cdot da_{11'}a_{33'} = 0,\\
(AD)'' &\equiv m_{ad}'' \cdot ad_{24'}d_{2'4} - n_{ad}'' \cdot da_{44'}a_{22'} = 0,\\
(AC)' &\equiv m_{ac}' \cdot ac_{14'}c_{1'4} - n_{ac}' \cdot ca_{11'}a_{44'} = 0,\\
(AC)'' &\equiv m_{ac}'' \cdot ac_{32'}c_{3'2} - n_{ac}'' \cdot ca_{22'}a_{33'} = 0,\\
(BD)' &\equiv m_{bd}' \cdot bd_{13'}d_{2'4} - n_{bd}' \cdot db_{43'}b_{12'} = 0,\\
(BD)'' &\equiv m_{bd}'' \cdot bd_{24'}d_{1'3} - n_{bd}'' \cdot db_{34'}b_{21'} = 0.
\end{aligned}
\tag{6}
$$

Consider now the two equations generated by the two projectivities $(AB)'$ and $(AC)'$, characterized by the common vertex A. Writing the two equations in the form $\phi' - \psi' = 0$, $\phi'' - \psi'' = 0$, we get the equation $\phi'\psi'' - \phi''\psi' = 0$. This sextic is necessarily reducible, and contains the given C_3 as a part and some other cubic $C_3{}^*$. From the equations, $(AB)'$ and $(AC)'$, it is apparent that a and $a_{11'}$ are parts of $C_3{}^*$, so that, after dropping a and $a_{11'}$, there remains the quartic

$$
(AB)'(AC)' \equiv m_{ac}'n_{ab}' \cdot ba_{22'}c_{14'}c_{1'4} - m_{ab}'n_{ac}' \cdot ca_{44'}b_{12'}b_{1'2} = 0. \tag{7}
$$

By the symbol $(AB)'(AC)'$ the process is indicated by which this quartic was obtained. This quartic contains the given C_3 as a part so that the remainder is a line. From (7) it is seen that it passes through the four points of intersection of $b_{12'}$ and $c_{1'4}$ (symbolically $(b_{12'}c_{1'4})$); similarly through $(b_{1'2}c_{14'})$, $(ba_{44'})$, $(ca_{22'})$, which are not on the cubic. These four points are therefore collinear. In a similar manner we obtain eleven other quartics, each containing the given C_3 and a line, represented by the processes $(AC)'(AD)'$, $(AD)'(AB)'$, $(AB)'(AC)''$, $(AC)''(AD)''$, $(AD)''(AB)'$, $(AB)''(AC)''$, $(AC)''(AD)'$, $(AD)'(AB)''$, $(AB)''(AC)'$, $(AC)'(AD)''$, $(AD)''(AB)''$. In this manner, keeping A fixed, we obtain 12 lines. Likewise for each of the other points B, C, D of the Steinerian quadruple 12 other lines are obtained. From the three equations

$(AB)'$, $(AC)'$, $(AD)'$ in (6), and the cyclic arrangement of the processes $(AB)'(AC)'$, $(AC)'(AD)'$, $(AD)'(AB)'$ we get the three quartics:

$$\begin{aligned} &(AB)'(AC)' \equiv m_{ac}'n_{ab}' \cdot ba_{22'}c_{14'}c_{1'4} - m_{ab}'n_{ac}' \cdot ca_{44'}b_{12'}b_{1'2} = 0,\\ (8)\quad &(AC)'(AD)' \equiv m_{ad}'n_{ac}' \cdot ca_{44'}d_{13'}d_{1'3} - m_{ac}'n_{ad}' \cdot da_{33'}c_{14'}c_{1'4} = 0,\\ &(AD)'(AB)' \equiv m_{ab}'n_{ad}' \cdot da_{33'}b_{12'}b_{1'2} - m_{ad}'n_{ab}' \cdot ba_{22'}d_{13'}d_{1'3} = 0. \end{aligned}$$

When the first, second and third of these equations in the same order are multiplied by $n_{ad}' \cdot da_{33'}$, $n_{ab}' \cdot ba_{22'}$, $n_{ac}' \cdot ca_{44'}$, and added, their sum vanishes identically. Denoting the polynomial representing the given cubic C_3 by ϕ and the lines contained in the quartics (8) in the same order by the symbolic linear expressions α, β, γ, we have, after this multiplication and addition,

$$(n_{ad}' \cdot da_{33'}\alpha + n_{ab}' \cdot ba_{22'}\beta + n_{ac}' \cdot ca_{44'}\gamma)\phi \equiv 0$$

identically. But since $\phi \not\equiv 0$, we necessarily have

$$(9)\qquad n_{ad}' \cdot da_{33'}\alpha + n_{ab}' \cdot ba_{22'}\beta + n_{ac}' \cdot ca_{44'}\gamma \equiv 0.$$

From (8) it is easily seen that on each of the lines α, β, γ are four collinear points; on

$$(10)\qquad \begin{array}{lllll} \alpha: & (b_{12'}c_{1'4}), & (b_{1'2}c_{14'}), & (ba_{44'}), & (ca_{22'});\\ \beta: & (c_{14'}d_{1'3}), & (c_{1'4}d_{13'}), & (ca_{33'}), & (da_{44'});\\ \gamma: & (d_{13'}b_{1'2}), & (d_{1'3}b_{12'}), & (da_{22'}), & (ba_{33'}). \end{array}$$

In (9) the point of intersection of α and β does not lie on $a_{44'}$, since, according to (10), α cuts $a_{44'}$ in $(ba_{44'})$, and β cuts $a_{44'}$ in $(da_{44'})$. Neither does it lie on c, since α cuts c in $(ca_{22'})$, and β cuts c in $(ca_{33'})$. Hence, as (9) vanishes identically, the point of intersection of α and β must lie on γ; i. e., α, β and γ are concurrent.

It is noticed that each of the three equations represented by $(AB)'$, $(AC)'$, $(AD)'$ from which the three lines (10) were obtained had the line $a_{11'}$ in its structure. The signature 11′ may therefore be considered as characteristic of these concurrent lines. In a similar manner, with the signatures of all other of the 16 lines, different from a, b, c, d, which with these determine the six quadratic projectivities, are associated three concurrent lines, so that altogether again the 48 lines, determined above, result.

3. Explicit representation of the configuration of the 48 lines by means of the group of order 16. Keeping in mind that to the letters a, b, c, d are attached in the same order the signatures 11′, 22′, 33′, 44′; 12′, 1′2, 34′,

$3'4$; $14'$, $1'4$, $23'$, $2'3$; $13'$, $1'3$, $24'$, $2'4$ as subscripts, the complete table of the 48 lines is obtained by applying the 16 substitutions of the group (3) to the three lines with the signature $11'$, which shall be represented by the identical substitution. In this table we write below each substitution, the signature of the three concurrent lines, the processes by which they were obtained and the four points on each of the three lines.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 1' & 2' & 3' & 4' \\ 1 & 2 & 3 & 4 & 1' & 2' & 3' & 4' \end{pmatrix},$$

$$11' \left\{ \begin{array}{lllll} (AB)'(AC)': & (b_{12'}c_{1'4}), & (b_{1'2}c_{14'}), & (ba_{44'}), & (ca_{22'}), \\ (AC)'(AD)': & (c_{14'}d_{1'3}), & (c_{1'4}d_{13'}), & (ca_{33'}), & (da_{44'}), \\ (AD)'(AB)': & (d_{13'}b_{1'2}), & (d_{1'3}b_{12'}), & (da_{22'}), & (ba_{33'}); \end{array} \right.$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 1' & 2' & 3' & 4' \\ 2 & 1 & 4 & 3 & 2' & 1' & 4' & 3' \end{pmatrix},$$

$$22' \left\{ \begin{array}{lllll} (AB)'(AC)'': & (b_{21'}c_{2'3}), & (b_{2'1}c_{23'}), & (ba_{33'}), & (ca_{11'}), \\ (AC)''(AD)'': & (c_{23'}d_{2'4}), & (c_{2'3}d_{24'}), & (ca_{44'}), & (da_{33'}), \\ (AD)''(AB)': & (d_{24'}b_{2'1}), & (d_{2'4}b_{21'}), & (da_{11'}), & (ba_{44'}); \end{array} \right.$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 1' & 2' & 3' & 4' \\ 3 & 4 & 1 & 2 & 3' & 4' & 1' & 2' \end{pmatrix},$$

$$33' \left\{ \begin{array}{lllll} (AB)''(AC)'': & (b_{34'}c_{3'2}), & (b_{3'4}c_{32'}), & (ba_{22'}), & (ca_{44'}), \\ (AC)''(AD)': & (c_{32'}d_{3'1}), & (c_{3'2}d_{31'}), & (ca_{11'}), & (da_{22'}), \\ (AD)'(AB)'': & (d_{31'}b_{3'4}), & (d_{3'1}b_{34'}), & (da_{44'}), & (ba_{11'}); \end{array} \right.$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 1' & 2' & 3' & 4' \\ 4 & 3 & 2 & 1 & 4' & 3' & 2' & 1' \end{pmatrix},$$

$$44' \left\{ \begin{array}{lllll} (AB)'(AC)': & (b_{43'}c_{4'1}), & (b_{4'3}c_{41'}), & (ba_{11'}), & (ca_{33'}), \\ (AC)'(AD)'': & (c_{41'}d_{4'2}), & (c_{4'1}d_{42'}), & (ca_{22'}), & (da_{11'}), \\ (AD)''(AB)': & (d_{42'}b_{4'3}), & (d_{4'2}b_{43'}), & (da_{33'}), & (ba_{22'}); \end{array} \right.$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 1' & 2' & 3' & 4' \\ 1 & 2 & 3 & 4 & 2' & 1' & 4' & 3' \end{pmatrix},$$

$$12' \left\{ \begin{array}{lllll} (BA)'(BD)': & (a_{11'}d_{2'4}), & (a_{22'}d_{13'}), & (ab_{43'}), & (db_{21'}), \\ (BD)'(BC)': & (d_{13'}c_{2'3}), & (d_{2'4}c_{14'}), & (db_{34'}), & (cb_{43'}), \\ (BC)'(BA)': & (c_{14'}a_{22'}), & (c_{2'3}a_{11'}), & (cb_{21'}), & (ab_{34'}); \end{array} \right.$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 1' & 2' & 3' & 4' \\ 2 & 1 & 4 & 3 & 1' & 2' & 3' & 4' \end{pmatrix},$$

$$21' \left\{ \begin{array}{lllll} (BA)'(BD)'': & (a_{22'}d_{1'3}), & (a_{1'1}d_{24'}), & (ab_{34'}), & (db_{12'}), \\ (BD)''(BC)'': & (d_{24'}c_{1'4}), & (d_{1'3}c_{23'}), & (db_{43'}), & (cb_{34'}), \\ (BC)''(BA)': & (c_{23'}a_{1'1}), & (c_{1'4}a_{22'}), & (cb_{12'}), & (ab_{43'}); \end{array} \right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\3&4&1&2&4'&3'&2'&1'\end{pmatrix},$$

$$34'\left\{\begin{array}{lllll}(BA)''(BD)': & (a_{33'}d_{4'2}), & (a_{4'4}d_{31'}), & (ab_{21'}), & (db_{43'}),\\ (BD)'(BC)': & (d_{31'}c_{4'1}), & (d_{4'2}c_{32'}), & (db_{12'}), & (cb_{21'}),\\ (BC)'(BA)'': & (c_{32'}a_{4'4}), & (c_{4'1}a_{33'}), & (cb_{43'}), & (ab_{12'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\4&3&2&1&3'&4'&1'&2'\end{pmatrix},$$

$$43'\left\{\begin{array}{lllll}(BA)'(BD)': & (a_{44'}d_{3'1}), & (a_{3'3}d_{42'}), & (ab_{12'}), & (db_{34'}),\\ (BD)'(BC)'': & (d_{42'}c_{3'2}), & (d_{3'1}c_{41'}), & (db_{21'}), & (cb_{12'}),\\ (BC)''(BA)': & (c_{41'}a_{3'3}), & (c_{3'2}a_{44'}), & (cb_{34'}), & (ab_{21'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\1&2&3&4&4'&3'&2'&1'\end{pmatrix},$$

$$14'\left\{\begin{array}{lllll}(CD)'(CA)': & (d_{13'}a_{4'4}), & (d_{4'2}a_{11'}), & (dc_{41'}), & (ac_{23'}),\\ (CA)'(CB)': & (a_{11'}b_{4'3}), & (a_{4'4}b_{12'}), & (ac_{32'}), & (bc_{41'}),\\ (CB)'(CD)': & (b_{12'}d_{4'2}), & (b_{4'3}d_{13'}), & (bc_{23'}), & (dc_{32'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\4&3&2&1&1'&2'&3'&4'\end{pmatrix},$$

$$41'\left\{\begin{array}{lllll}(CD)'(CA)': & (d_{42'}a_{1'1}), & (d_{1'3}a_{44'}), & (dc_{14'}), & (ac_{32'}),\\ (CA)'(CB)'': & (a_{44'}b_{1'2}), & (a_{1'1}b_{43'}), & (ac_{23'}), & (bc_{14'}),\\ (CB)''(CD)': & (b_{43'}d_{1'3}), & (b_{1'2}d_{42'}), & (bc_{32'}), & (dc_{23'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\2&1&4&3&3'&4'&1'&2'\end{pmatrix},$$

$$23'\left\{\begin{array}{lllll}(CD)'(CA)'': & (d_{24'}a_{3'3}), & (d_{3'1}a_{22'}), & (dc_{32'}), & (ac_{14'}),\\ (CA)''(CB)'': & (a_{22'}b_{3'4}), & (a_{3'3}b_{21'}), & (ac_{41'}), & (bc_{32'}),\\ (CB)''(CD)': & (b_{21'}d_{3'1}), & (b_{3'4}d_{24'}), & (bc_{14'}), & (dc_{41'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\3&4&1&2&2'&1'&4'&3'\end{pmatrix},$$

$$32'\left\{\begin{array}{lllll}(CD)''(CA)'': & (d_{31'}a_{22'}), & (d_{2'4}a_{33'}), & (dc_{23'}), & (ac_{41'}),\\ (CA)''(CB)': & (a_{33'}b_{2'1}), & (a_{2'2}b_{34'}), & (ac_{14'}), & (bc_{23'}),\\ (CB)'(CD)'': & (b_{34'}d_{2'4}), & (b_{2'1}d_{31'}), & (bc_{41'}), & (dc_{14'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\1&2&3&4&3'&4'&1'&2'\end{pmatrix},$$

$$13'\left\{\begin{array}{lllll}(DC)'(DB)': & (c_{14'}b_{3'4}), & (c_{3'2}b_{12'}), & (cd_{42'}), & (bd_{24'}),\\ (DB)'(DA)': & (b_{12'}a_{3'3}), & (b_{3'4}a_{11'}), & (bd_{31'}), & (ad_{42'}),\\ (DA)'(DC)': & (a_{11'}c_{3'2}), & (a_{3'3}c_{14'}), & (ad_{24'}), & (cd_{31'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\3&4&1&2&1'&2'&3'&4'\end{pmatrix},$$

$$31' \left\{\begin{array}{lllll}(DC)''(DB)'': & (c_{32'}b_{1'2}), & (c_{1'4}b_{34'}), & (cd_{24'}), & (bd_{42'}),\\ (DB)''(DA)': & (b_{34'}a_{1'1}), & (b_{1'2}a_{33'}), & (bd_{13'}), & (ad_{24'}),\\ (DA)'(DC)'': & (a_{33'}c_{1'4}), & (a_{1'1}c_{32'}), & (ad_{42'}), & (cd_{13'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\2&1&4&3&4'&3'&2'&1'\end{pmatrix},$$

$$24' \left\{\begin{array}{lllll}(DC)'(DB)'': & (c_{23'}b_{4'3}), & (c_{4'1}b_{21'}), & (cd_{31'}), & (bd_{13'}),\\ (DB)''(DA)'': & (b_{21'}a_{4'4}), & (b_{4'3}a_{22'}), & (bd_{42'}), & (ad_{31'}),\\ (DA)''(DC)': & (a_{22'}c_{4'1}), & (a_{4'4}c_{23'}), & (ad_{13'}), & (cd_{42'});\end{array}\right.$$

$$\begin{pmatrix}1&2&3&4&1'&2'&3'&4'\\4&3&2&1&2'&1'&4'&3'\end{pmatrix},$$

$$42' \left\{\begin{array}{lllll}(DC)''(DB)': & (c_{41'}b_{2'1}), & (c_{2'3}b_{43'}), & (cd_{13'}), & (bd_{31'}),\\ (DB)'(DA)'': & (b_{43'}a_{2'2}), & (b_{2'1}a_{44'}), & (bd_{24'}), & (ad_{13'}),\\ (DA)''(DC)': & (a_{44'}c_{2'3}), & (a_{2'2}c_{41'}), & (ad_{31'}), & (cd_{24'});\end{array}\right.$$

4. Discussion of the configuration. The foregoing table contains 96 distinct points through which the 48 lines pass. Through each of these points two of the lines pass. The 48 lines in 16 sets of three are concurrent, so that they pass through 16 other fixed points in triplets. Of the 96 points 48 are formed by intersections of the 16 lines forming the projectivities. The other 48 points are formed by the intersections of the tangents a, b, c, d with the same 16 lines. In the figure only one of the sixteen sets of concurrent lines is shown; it corresponds to the set with the signature 43′ and the three lines are denoted by α_1, β_1, γ_1. The remaining fifteen sets may also be constructed without difficulty. They have been omitted in order not to overload the figure.

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